Calculus AB

3-7

Optimization Problems

Find two positive numbers that satisfy the given requirements. (pg 223)

4) The product is 192 and the sum is a minimum.

Let $x =$ 1st number $\boxed{8\sqrt{3}}$

$\frac{192}{x} =$ 2nd number $\boxed{8\sqrt{3}}$

$S(x) = x + \frac{192}{x} = x + 192x^{-1}$

$S'(x) = 1 - \frac{192}{x^2}$

$0 = 1 - \frac{192}{x^2}$

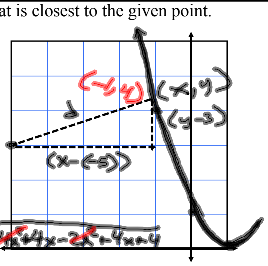
$\frac{192}{x^2} = 1$

$\sqrt{192} = \sqrt{x^2} \rightarrow |x| = 8\sqrt{3}$

$x = \pm 8\sqrt{3}$

Find the point on the graph of the function that is closest to the given point.

13) $\overset{y}{f(x)} = (x - 1)^2$ (-5, 3)



$d = \sqrt{(x+5)^2 + (y-3)^2}$

$d = \sqrt{(x+5)^2 + [(x-1)^2 - 3]^2}$

$d = \sqrt{x^2 + 10x + 25 + [x^2 - 2x - 2]^2}$

$d = \sqrt{x^2 + 10x + 25 + \cancel{x^4} - \cancel{2x^3} - \cancel{2x^2} - \cancel{2x^3} + \cancel{4x^2} + 4x - \cancel{2x^2} + 4x + 4}$

$= \sqrt{x^4 - 4x^3 + x^2 + 18x + 29}$

$\frac{dd}{dx} = \frac{\overset{2}{\cancel{4}}x^3 \overset{-6}{-12x^2} \overset{+1}{+2x} \overset{+9}{+18}}{\cancel{2}\sqrt{x^4 - 4x^3 + x^2 + 18x + 29}}$

$0 = 2x^3 - 6x^2 + x + 9$

c.p. $x = -1$ (From calc)

min by 1st deriv test

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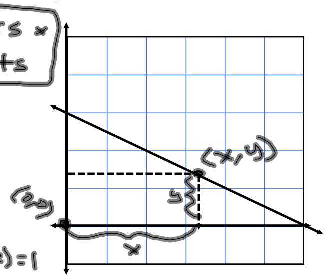
26) A rectangle is bounded by the x- and y- axes and the graph of $y = \frac{(6 - x)}{2}$. What length and width should the rectangle have so that its area is a maximum?

$\boxed{\begin{array}{l} 3 \text{ units } x \\ \frac{3}{2} \text{ units} \end{array}}$



$A = xy$

$A(x) = \frac{x(6-x)}{2}$

$A(x) = 3x - \frac{1}{2}x^2$

$A'(x) = 3 - x$

c.p. at $x = 3$ and a max

$A'(2) = 1$

$A'(3) = 0$

$A'(4) = -1$

Assignments: Pg. 223

Day 1

3-25 odd

Day 2

29, 33, 34, 35, 43, 45, 47, 49, 54